

# Higher order fluctuations of conserved charges in heavy ion collisions

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## Abstract.

We investigate the effect of the global charge conservation on the cumulants of conserved charges in relativistic heavy ion collisions in a finite rapidity window by studying the time evolution of cumulants in the hadronic medium. It is argued that the global charge conservation does not affect the experimental result of the net-electric charge fluctuation observed by ALICE, because of the finite diffusion distance of charged particles in the hadronic stage.

## 1. Introduction

Bulk fluctuations of conserved charges measured by event-by-event analyses in relativistic heavy ion collisions, especially higher order cumulants, are expected to be promising observables to characterize properties of the hot medium created by collision events and to find the QCD critical point [1, 2, 3]. Recently, experimental investigations of these observables in heavy ion collisions have been actively performed at the RHIC [4] and LHC [5].

One of the important properties of these fluctuations in heavy ion collisions is that experimentally-observed fluctuations had experienced non-equilibrium evolution, because of the dynamical nature of the hot medium created in heavy ion collisions [3, 6]. In fact, the recent experimental result on the rapidity window dependence of net electric charge fluctuation by the ALICE Collaboration at the LHC [5] shows that the value of the second order cumulant of the net electric charge is significantly suppressed compared with the one in the equilibrated hadronic medium. This result is reasonably understood if one interprets the suppression as a sign of the survival of the small fluctuation created in the primordial deconfined medium [2, 3, 6].

However, there exists another mechanism called the global charge conservation (GCC) to cause the suppression of fluctuations compared with the equilibrated value. If one counts a conserved charge in the total system generated by experiments, there are no event-by-event fluctuations because of the charge conservation. This fact is referred to as the GCC [1]. In Ref. [7], on the assumption that the equilibration is established in the final state of the heavy ion collision, the magnitude of this effect is estimated. It, however, should be noted that the suppression of charge fluctuation observed at ALICE [5] is stronger than the one estimated in Ref. [7]. This result shows that there exists another contribution for the suppression in addition to the GCC, such as the nonequilibrium effect as originally addressed in Ref. [2]. The effect of the GCC at LHC energy, therefore, has to be discussed again with the nonthermal effects incorporated.

In the present study, we investigate the effect of the GCC on cumulants of conserved charges under such nonequilibrium circumstances, by describing the time evolution of fluctuations in a finite volume system [8]. We also discuss the effects of the GCC on higher-order cumulants of conserved charges for the first time. By comparing our result with the experimental one at ALICE in Ref. [5], we find that the effect of GCC on the net electric charge fluctuation in the rapidity window observed by this experiment is almost negligible.

## 2. Stochastic formalism to describe diffusion of hadrons

In heavy ion collisions with sufficiently large  $\sqrt{s_{\text{NN}}}$ , the hot medium created at the mid-rapidity has an approximate boost invariance. We denote the net number of a conserved charge per unit space-time rapidity  $\eta$  as  $n(\eta, \tau)$  with the proper time  $\tau$ , and set  $\tau = \tau_0$  at hadronization. Because of the local charge conservation, the probability distribution of  $n(\eta, \tau_0)$  at hadronization inherits from the one that existed in the deconfined medium [1]. After hadronization, particles diffuse and rescatter, and the distribution of  $n(\eta, \tau)$  continues to approach the one of the equilibrated hadronic medium until kinetic freezeout at  $\tau = \tau_{\text{fo}}$ .

In this study, to describe the time evolution of fluctuation of the particle number at mid-rapidity  $Q(\Delta\eta, \tau) = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta n(\eta, \tau)$  in hadronic medium, with the size of the rapidity window to count the charge number  $\Delta\eta$ , until  $\tau = \tau_{\text{fo}}$  we adopt the diffusion master equation [3, 8]. In this model, we divide the system with the total rapidity length  $\eta_{\text{tot}}$  into  $M$  discrete cells with an equal finite length  $a = \eta_{\text{tot}}/M$ . We then consider a single species of particles for the moment, and denote the particle number existing in the  $m$ th cell as  $n_m$  and the probability distribution that each cell contains  $n_m$  particles as  $P(\mathbf{n}, \tau)$  with  $\mathbf{n} = (n_0, n_1, \dots, n_m, \dots, n_{M-2}, n_{M-1})$ . Finally, we assume that each particle moves to the adjacent cells with a probability  $\gamma(\tau)$  per unit proper time, as a result of microscopic interactions. The probability  $P(\mathbf{n}, \tau)$  then obeys the diffusion master equation,

$$\partial_\tau P(\mathbf{n}, \tau) = \gamma(\tau) \sum_{m=0}^{M-1} [(n_m + 1)\{P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m+1}, \tau) + P(\mathbf{n} + \mathbf{e}_m - \mathbf{e}_{m-1}, \tau)\} - 2n_m P(\mathbf{n}, \tau)], \quad (1)$$

where  $\mathbf{e}_m$  is a unit vector whose all components are zero except for  $m$ th one, which takes unity.

The average and the Gaussian fluctuation of  $n(\eta, \tau)$  in Eq. (1) in the continuum limit,  $a \rightarrow 0$ , agree with the ones of the stochastic diffusion equation, [6]

$$\partial_\tau n(\eta, \tau) = D(\tau) \partial_\eta^2 n(\eta, \tau) + \partial_\eta \xi(\eta, \tau), \quad (2)$$

when one sets the diffusion coefficient as  $D(\tau) = \gamma(\tau)a^2$  [3]. Here,  $\xi(\eta, \tau)$  is the temporarily-local stochastic force. It is known that the fluctuation of Eq. (2) in equilibrium becomes of Gaussian [3, 8]. On the other hand, the diffusion master equation Eq. (1) can give rise to nonzero higher order cumulants in equilibrium [3]. This is the reason why we employ the diffusion master equation instead of Eq. (2).

## 3. Solution of diffusion master equation

Now, we determine the time evolution of cumulants by solving Eq. (1) and taking the continuum limit  $a \rightarrow 0$ . In order to take account of the finiteness of the size of the hot medium, we set reflecting boundaries at  $\eta = \pm\eta_{\text{tot}}/2$ . After some algebra [8], the cumulants of  $Q(\tau)$  with the fixed initial condition  $n(\eta, \tau_0) = N(\eta)$  are calculated to be

$$\langle (Q(\Delta\eta, \tau))^n \rangle_c = \int_{-\eta_{\text{tot}}/2}^{\eta_{\text{tot}}/2} d\eta N(\eta) H_n(\eta), \quad (3)$$

where  $H_2(\eta) = I(\eta) - I(\eta)^2$ ,  $H_4(\eta) = I(\eta) - 7I(\eta)^2 + 12I(\eta)^3 - 6I(\eta)^4$  with

$$I(\eta) = \frac{\Delta\eta}{\eta_{\text{tot}}} \sum_{k=-\infty}^{\infty} \cos\left(\frac{\pi k \eta}{\eta_{\text{tot}}}\right) \sin\left(\frac{\pi k \Delta\eta}{2\eta_{\text{tot}}}\right) \cos\left(\frac{\pi k}{2}\right) \exp\left[-\frac{1}{2}\left(\frac{\pi k d(\tau)}{\eta_{\text{tot}}}\right)^2\right] / \left(\frac{\pi k \Delta\eta}{2\eta_{\text{tot}}}\right). \quad (4)$$

Here,  $d(\tau) = \sqrt{2 \int_{\tau_0}^{\tau} d\tau' D(\tau')}$  is the average diffusion length of each Brownian particle.

Next, we extend the above results to the cases with general initial conditions with non-vanishing initial fluctuations. In the following, we also extend the formula to treat cumulants of the difference of densities of two particle species,  $n_1(\eta, \tau)$  and  $n_2(\eta, \tau)$ ,  $Q_{(\text{net})}(\Delta\eta, \tau) = \int_{-\Delta\eta/2}^{\Delta\eta/2} d\eta (n_1(\eta, \tau) - n_2(\eta, \tau))$ , in order to consider cumulants of conserved charges in QCD.

Next, let us constrain the initial condition. Because we consider a finite system, the initial condition at  $\tau = \tau_0$  should be determined in accordance with the GCC. In order to obtain the initial conditions which conform the GCC, here we model the initial configuration by an equilibrated free classical gas in a finite volume. In this system, one can obtain the correlation functions by taking the continuum limit of the multinomial distribution [8]. On the other hand, we assume that the total charge number at  $\tau = \tau_0$  in the total system obeys the one given by the grand canonical ensemble [3]. Using these initial conditions and Eq. (3), one obtains the higher order cumulants of  $Q_{(\text{net})}(\Delta\eta, \tau)$ .

#### 4. Effects of the GCC

Next, let us study how the GCC affects the rapidity window dependence of the cumulants of conserved charges. To describe the time evolution of cumulants, we use the dimensionless parameters  $T \equiv d(\tau)/\eta_{\text{tot}}$ . To make the argument simple, we consider the  $\Delta\eta$  dependence for the fixed initial condition, i.e., all fluctuations vanish at  $\tau = \tau_0$ . In Fig. 1, we show  $\langle(Q_{(\text{net})}(\Delta\eta))^n\rangle_c/\langle Q_{(\text{tot})}(\Delta\eta)\rangle_c$  for  $n = 2$  (left) and 4 (right), as a function of  $\Delta\eta/\eta_{\text{tot}}$  with the total length  $\eta_{\text{tot}}$  for five values of  $T$  including infinity. For comparison, we also show the result with an infinite volume by the dashed and dotted lines. The figure shows that  $\langle(Q_{(\text{net})}(\Delta\eta))^2\rangle_c$  vanishes at  $\Delta\eta/\eta_{\text{tot}} = 1$  for each  $T$ , namely  $\Delta\eta = \eta_{\text{tot}}$ , which is a trivial consequence of the GCC. On the other hand, as  $\Delta\eta$  becomes smaller from this value with fixed  $T$ , the result approaches the one with infinite volume. The effect of the GCC vanishes almost completely except for the range,

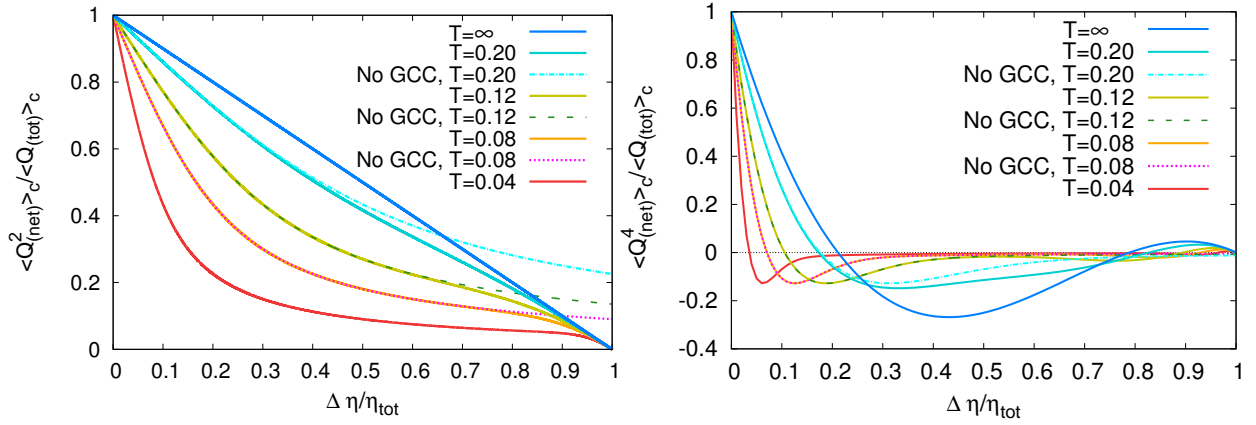
$$\frac{\eta_{\text{tot}} - \Delta\eta}{2} \lesssim d(\tau). \quad (5)$$

In this expression, the left-hand side is the distance between the left (right) boundary and the left (right) edge of the rapidity window, while the right-hand side is the diffusion length of each Brownian particle. When Eq. (5) is satisfied, particles which are reflected by one of the boundaries at least once can enter the rapidity window. Therefore, the existence of the boundaries can affect the fluctuations of conserved charges in the rapidity window. On the other hand, when the condition Eq. (5) is not satisfied, particles inside the rapidity window do not know the existence of the boundaries; in other words, the fact that the system is finite. In the latter case, therefore, the fluctuations in the rapidity window are free from the effect of the GCC.

With fixed  $\eta_{\text{tot}}$ , larger  $T$  corresponds to larger  $\tau$ . Figure 1(left) shows that as  $T$  becomes larger the fluctuation increases and approaches a linear function representing the equilibration, which is consistent with the result obtained in Ref. [7].

From Fig. 1 (right), which shows the fourth order cumulants for several values of  $T$ , one obtains completely the same conclusion on the effect of the GCC: The effect of the GCC is visible only for large values of  $\Delta\eta/\eta_{\text{tot}}$ .

In this analysis, it is assumed that the particle current vanishes on the two boundaries. This



**Figure 1.** Second (left panel) and fourth (right panel) order cumulants of conserved charges without initial fluctuation as a function of  $\Delta\eta/\eta_{\text{tot}}$  with five values of the parameter  $T$ . The corresponding results in an infinite volume are also plotted for several values of  $T$ .

assumption would not be suitable to describe the hot medium created by heavy ion collisions, since the hot medium does not have such hard boundaries but only baryon rich regions. From the above discussion, however, it is obvious that the effect of boundaries does not affect the fluctuations in the rapidity window unless the condition Eq. (5) is satisfied irrespective of the types of the boundaries.

We finally inspect the  $\Delta\eta$  dependence of the net electric charge fluctuations observed at ALICE [5] in more detail. In order to estimate the effect of the GCC, we must determine the magnitude of  $\eta_{\text{tot}}$ . From the pseudo-rapidity dependence of charged-particle yield at LHC energy,  $\sqrt{s_{\text{NN}}} = 2.76$  TeV, in Ref. [9], we take the value  $\eta_{\text{tot}} = 8$  in the following. With this  $\eta_{\text{tot}}$ , and the maximum rapidity coverage of the  $2\pi$  detector, TPC, of ALICE,  $\Delta\eta = 1.6$  [5], one obtains  $\Delta\eta/\eta_{\text{tot}} = 0.2$ . From Fig. 1, one can immediately conclude that the suppression of  $\langle(Q_{\text{net}}(\Delta\eta))^2\rangle_c$  in this experiment cannot be explained solely by the naive formula of the GCC [7]. From the discussion in the previous subsections, it is also concluded that the effect of the GCC on the diffusion in the hadronic stage is negligible in this experimental result.

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